

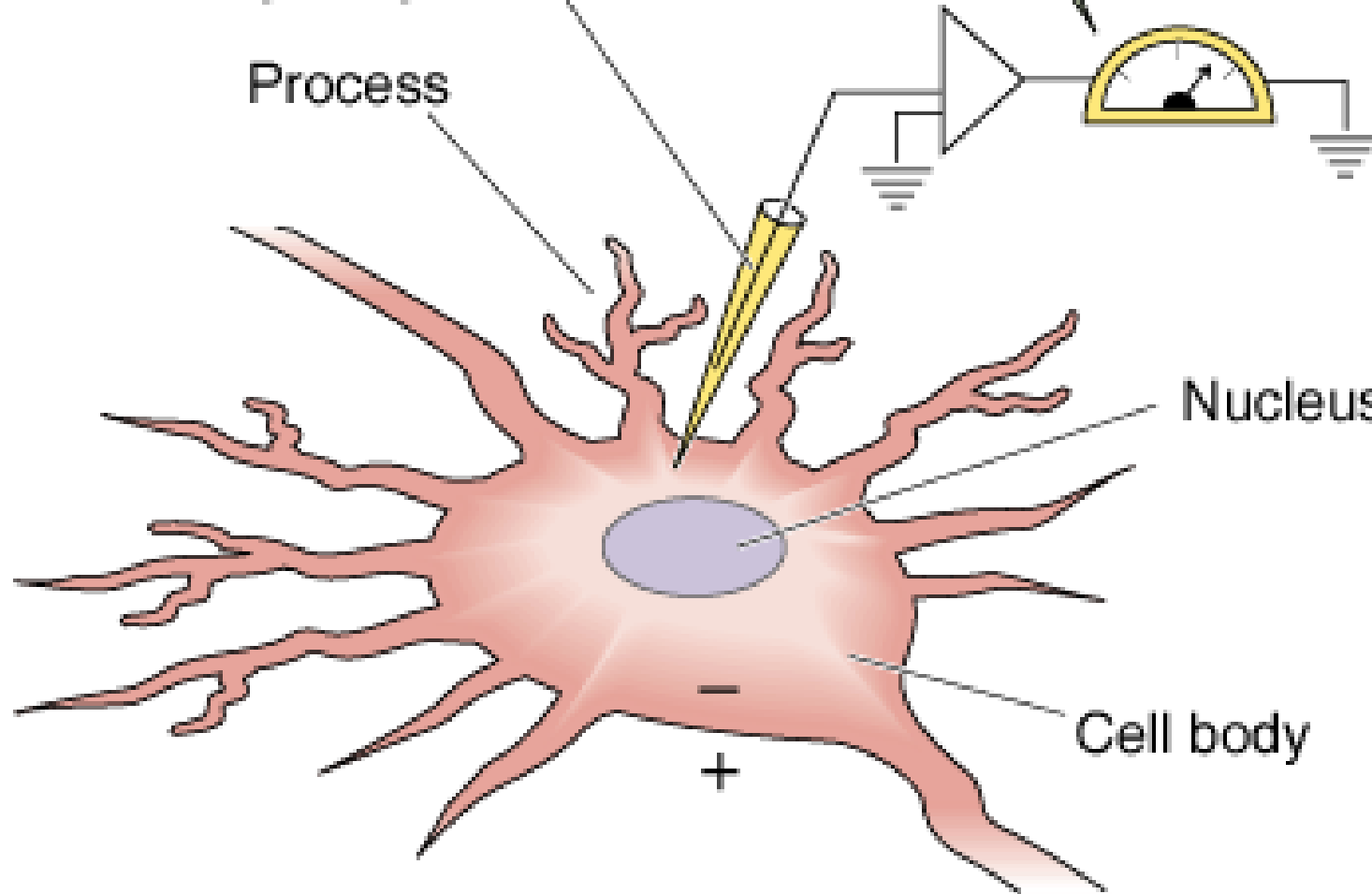
This voltmeter measures membrane potential.

Ag/AgCl in
3M KCl

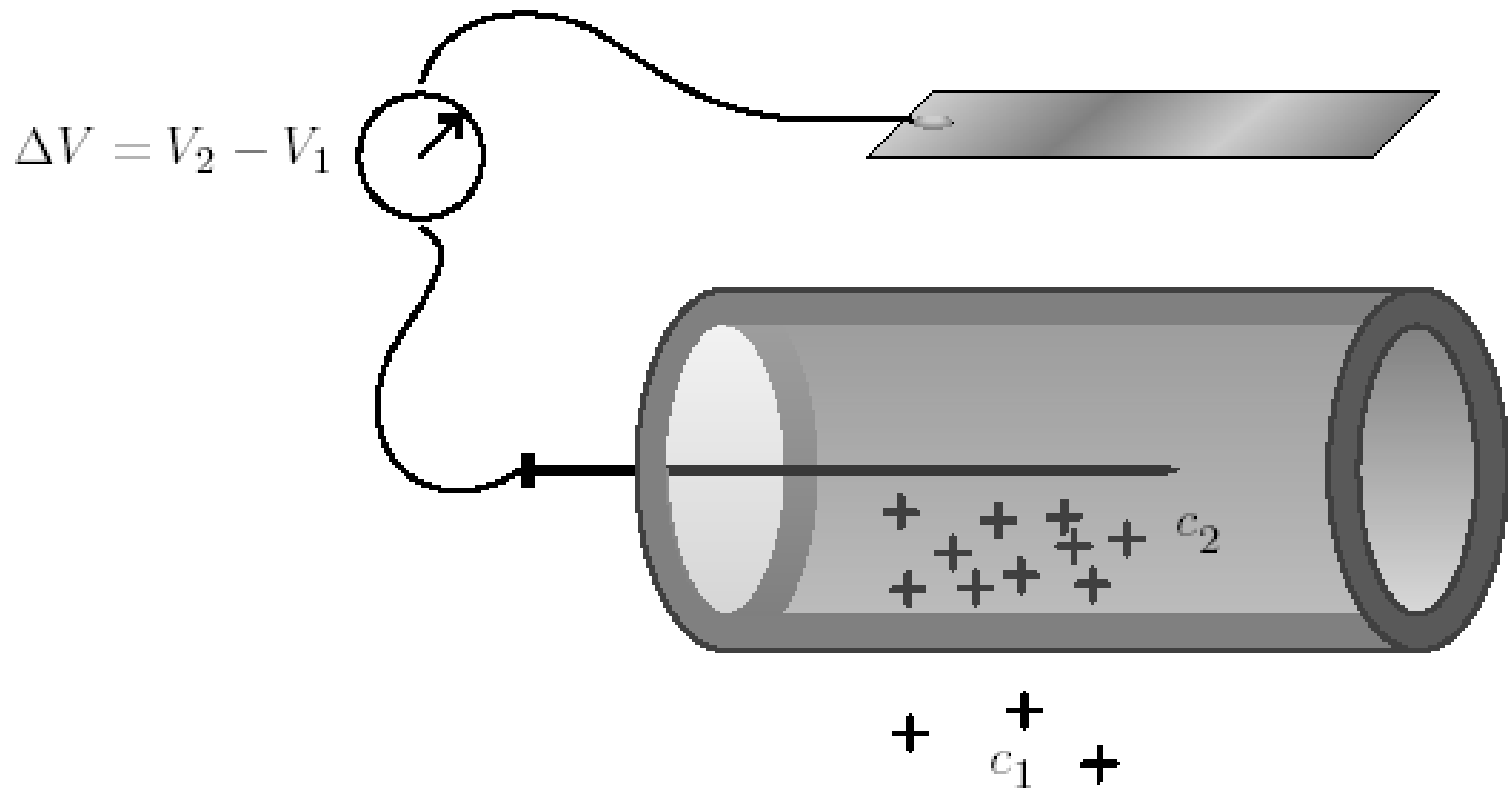
Process

Nucleus

Cell body

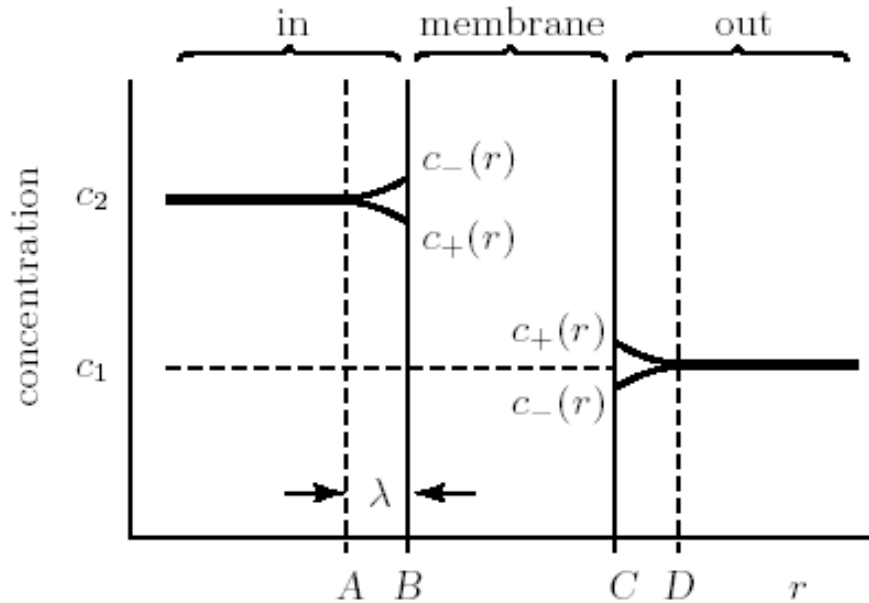


Ion	concentration (mM)	
	interstitial space	cell (“typical”)
Na ⁺	145	15
K ⁺	4.5	120
Ca ²⁺	1.2	1 x 10 ⁻⁷
Mg ²⁺	.55	1
Cl ⁻	116	20
HCO ₃ ⁻	25	15
glucose	5.9	low

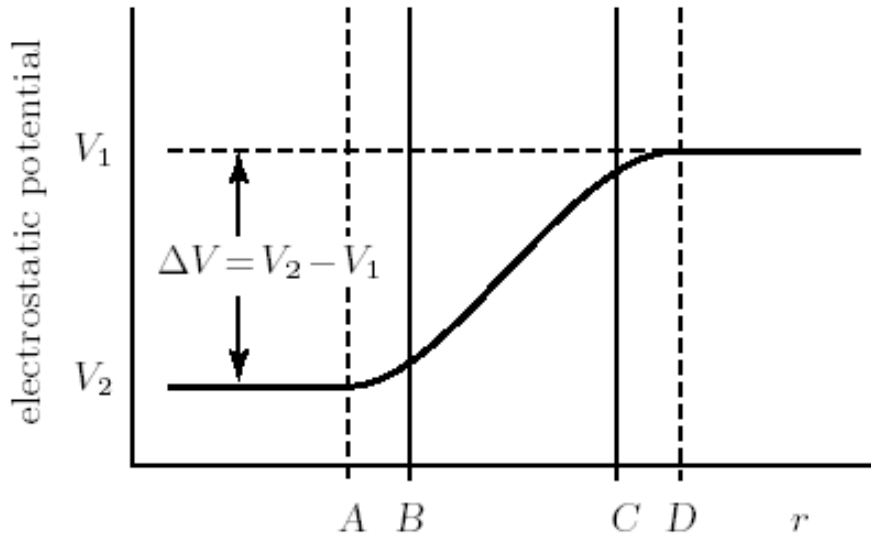


- Selective permeability to ions leads to membrane charge. Start with K^+ -permeant membrane, with abundance of K^+ inside the cell.

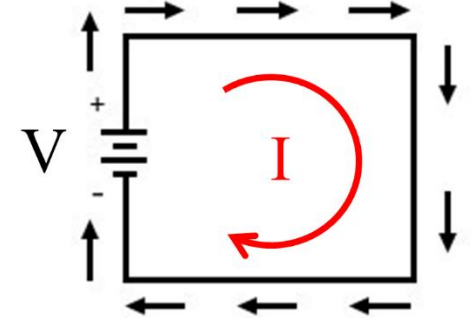
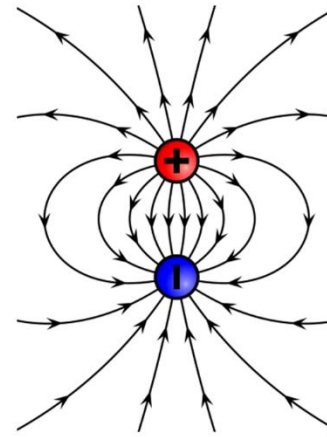
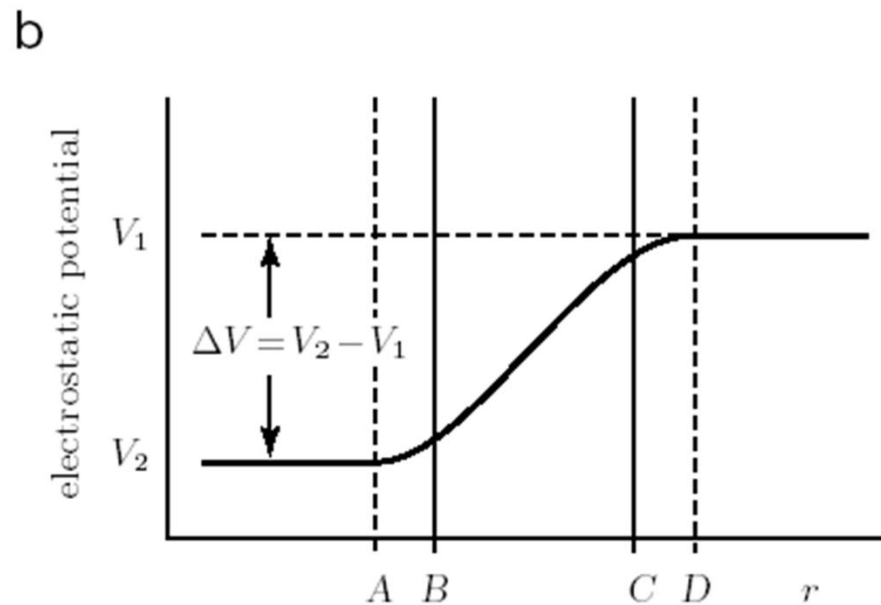
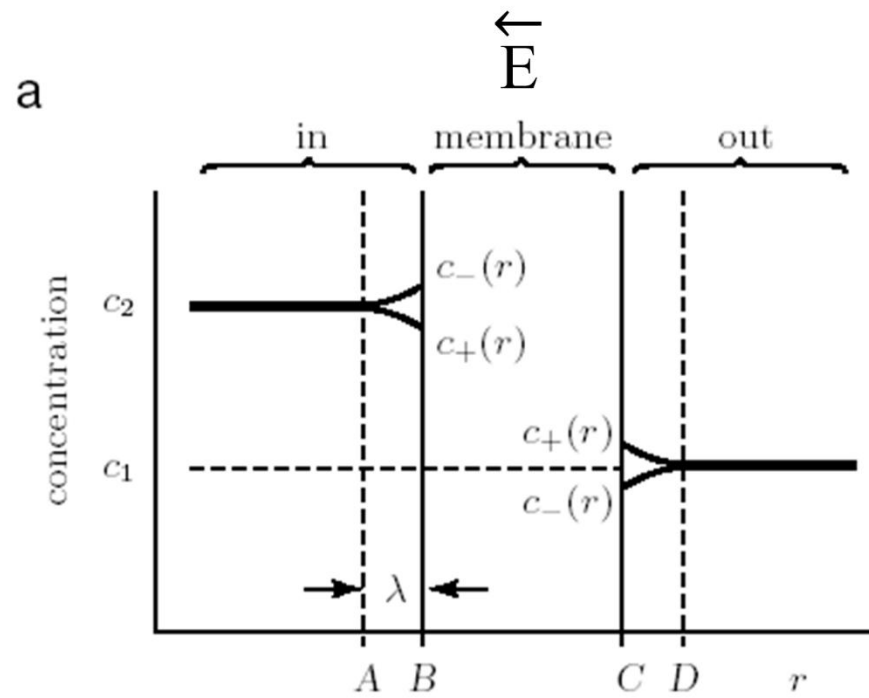
a



b



- Generation of a membrane voltage does not result in large-scale changes in ion concentration.
- Charge imbalance is localized to thin region near cell membrane



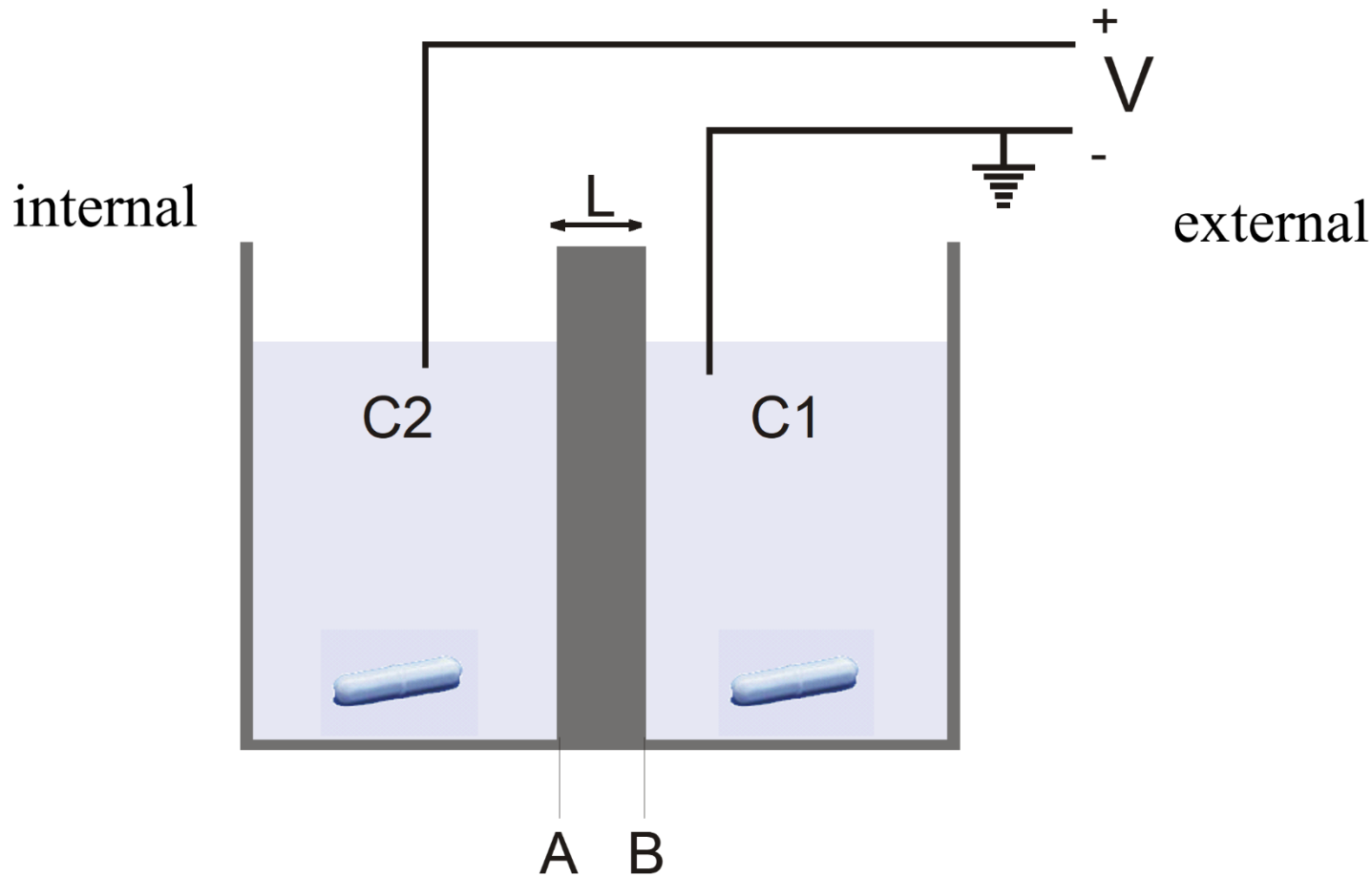
- Electric fields
 - Caused by charges
 - Indicate the direction that a positive charge would move
- Electric potentials
 - Integrates field

$$V = - \int E \cdot dl$$

Electrodiffusion

Motion of ions through a membrane is, as a starting point, a balance of response to chemical and electrical gradients

Start with transport through a region with partition coefficient of 1



Electrodiffusion

$$j(x) = -D \frac{dC(x)}{dx} + vC(x) \quad v = \mu * F = \mu * q * \varepsilon$$

ε = electric field

$$D = \mu k_B T$$

First, consider the case where $j(x)=0$

Assume that the electric field is present across the membrane, so a potential drop of V across some distance L .

This looks just like the sedimentation problem.

$$C(x) = C_0 * \exp(x*v/D) = C_0 * \exp(x*q*V/(k_B*T*L))$$

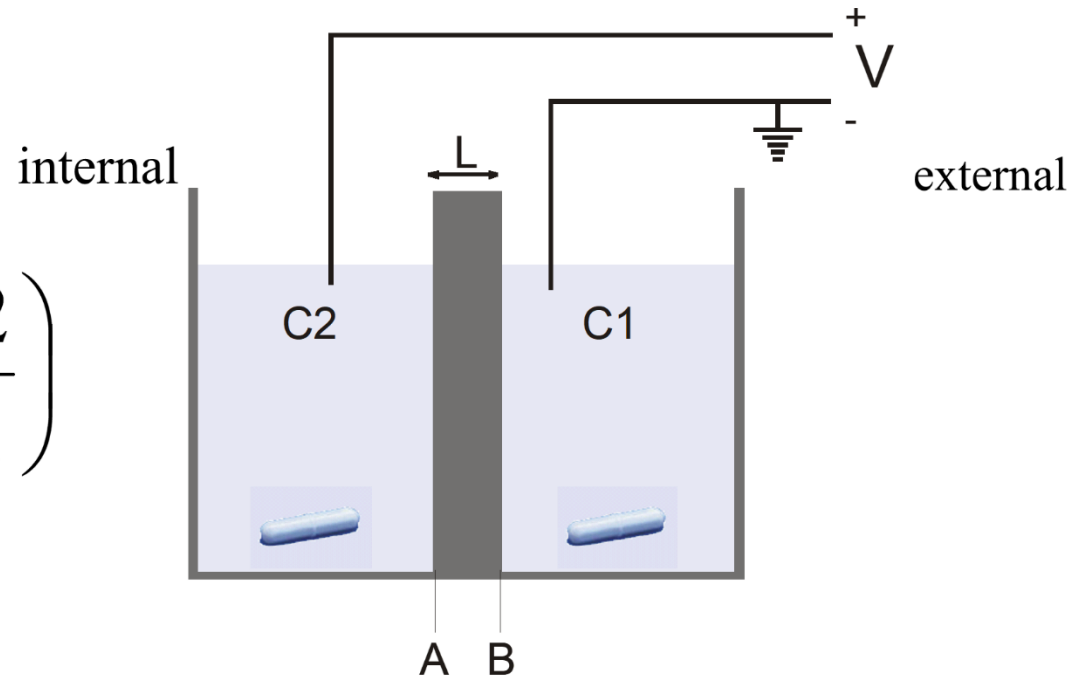
$$V^{Nernst} \equiv -\frac{k_B T}{q} \ln \left(\frac{C2}{C1} \right)$$

$C1$ = conc. ext.

$C2$ = conc. int.

Electrodiffusion

$$V^{Nernst} \equiv -\frac{k_B T}{q} \ln\left(\frac{C2}{C1}\right)$$



Ion	concentration (mM)		
	interstitial space	cell (“typical”)	V^{nernst} (mV) (37°C)
K^+ ,	4.5	120	-85

Electrodiffusion - GHK

$$j(x) = -D \frac{dC(x)}{dx} + vC(x) \quad v = \mu^* F = \mu^* q^* \varepsilon$$

$$\text{using } D = \mu k_B T$$

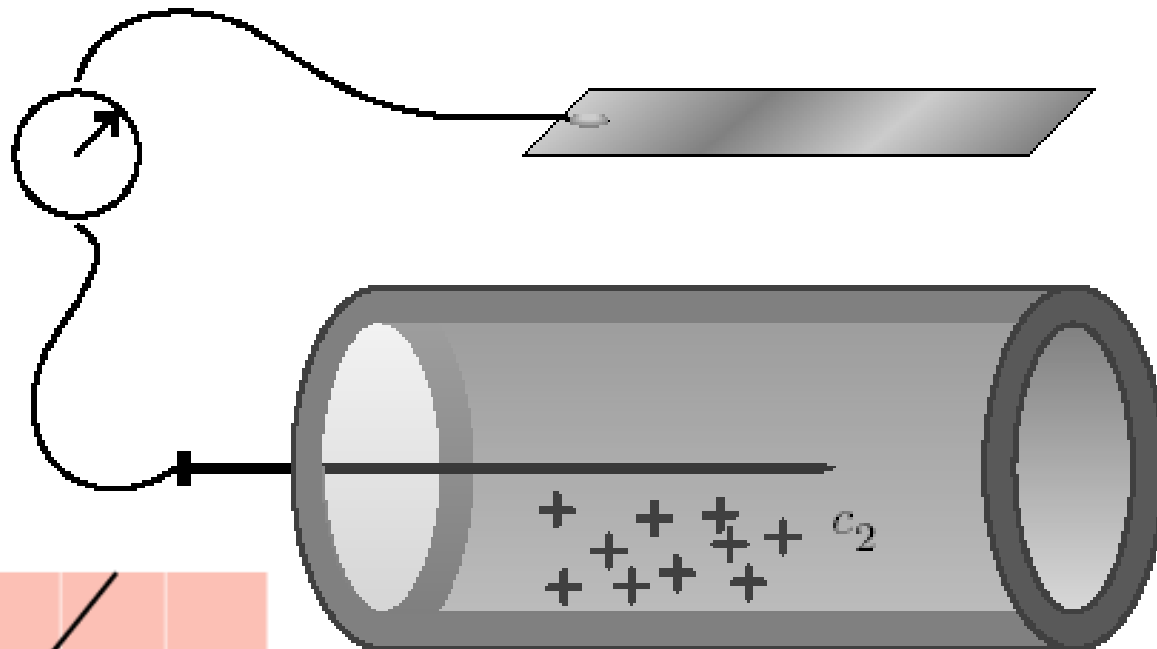
integrating and using boundary conditions: $C(0)=C_i$ and $C(L)=C_e$)

$$j = \frac{D}{L} \frac{qV}{k_B T} \frac{C_i - C_e \exp\left(-\frac{qV}{k_B T}\right)}{1 - \exp\left(-\frac{qV}{k_B T}\right)}$$

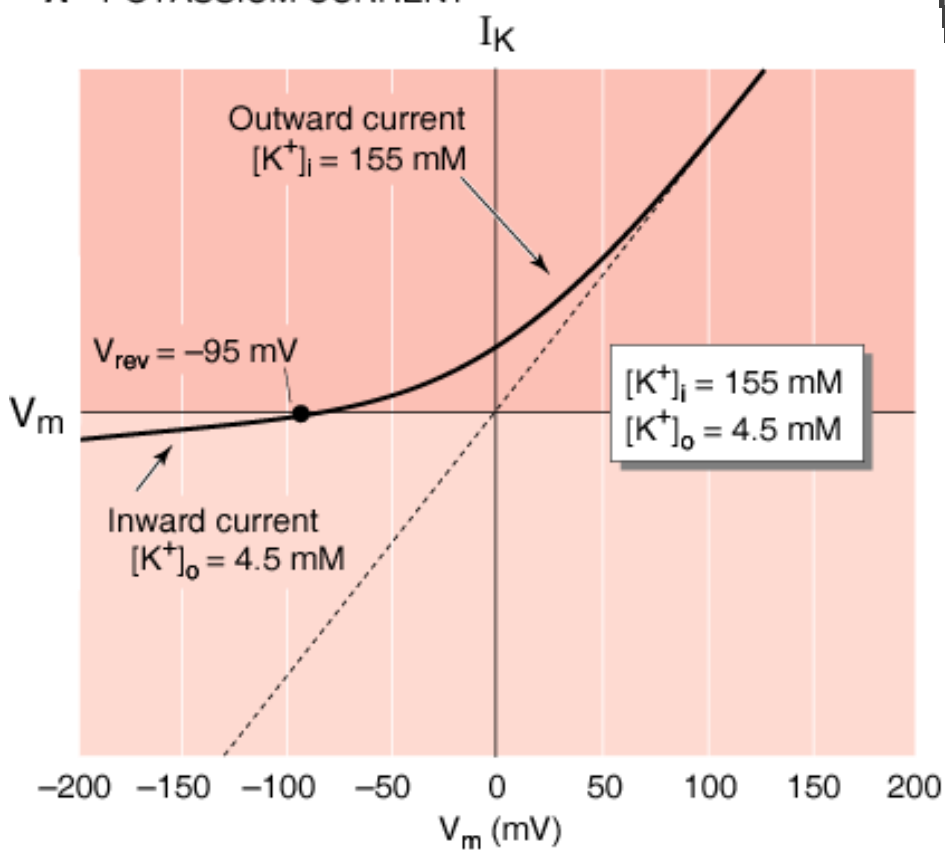
Goldman-Hodgkin-Katz eqn.

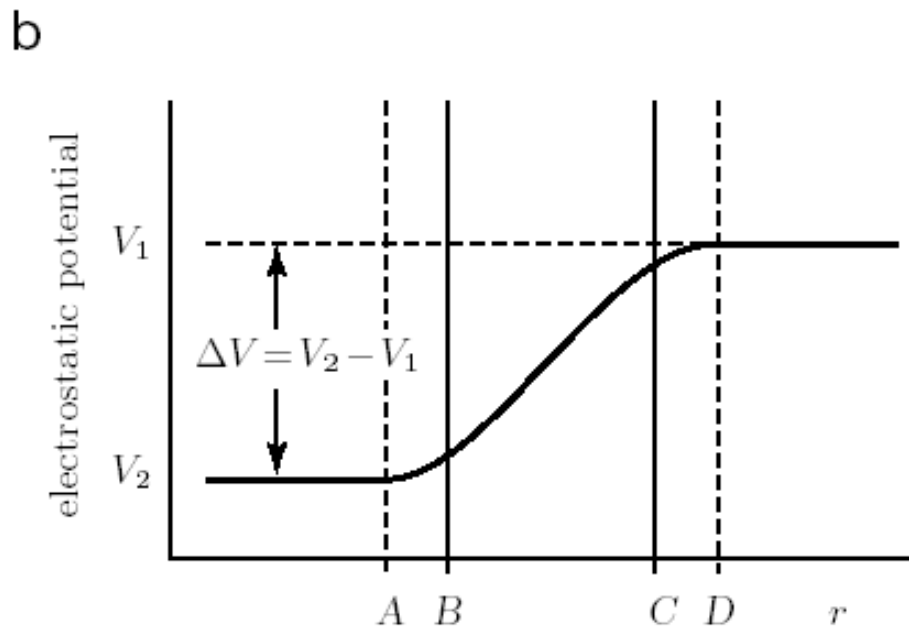
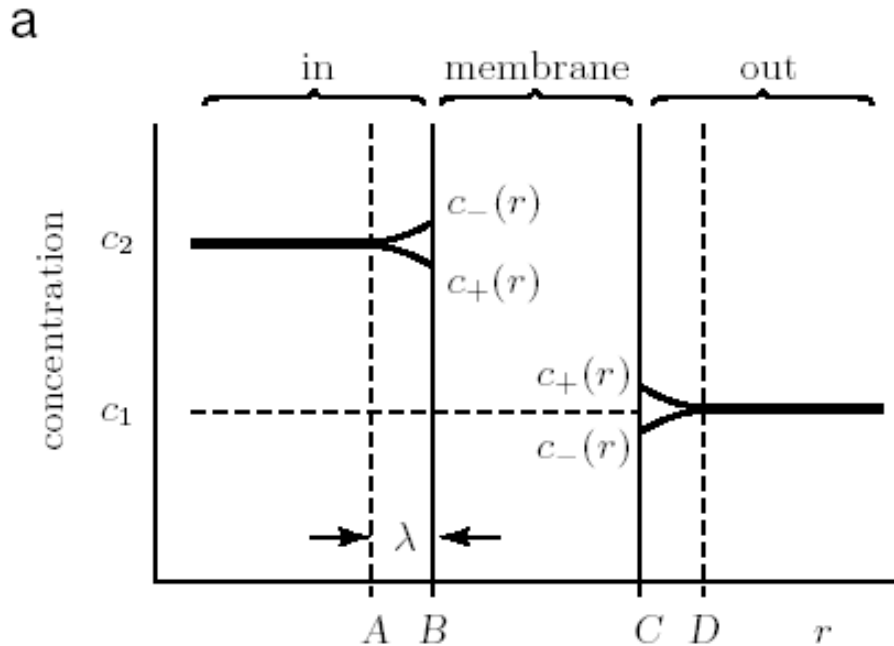
$$I = j^* \text{ charge} = j^* q^* N_A = \frac{D}{L} \frac{q^2 N_A V}{k_B T} \frac{C_i - C_e \exp\left(-\frac{qV}{k_B T}\right)}{1 - \exp\left(-\frac{qV}{k_B T}\right)}$$
$$= P \frac{q^2 N_A V}{k_B T} \frac{C_i - C_e \exp\left(-\frac{qV}{k_B T}\right)}{1 - \exp\left(-\frac{qV}{k_B T}\right)}; \quad P \text{ dependent on } D, L, \beta,$$

$$\Delta V = V_2 - V_1$$



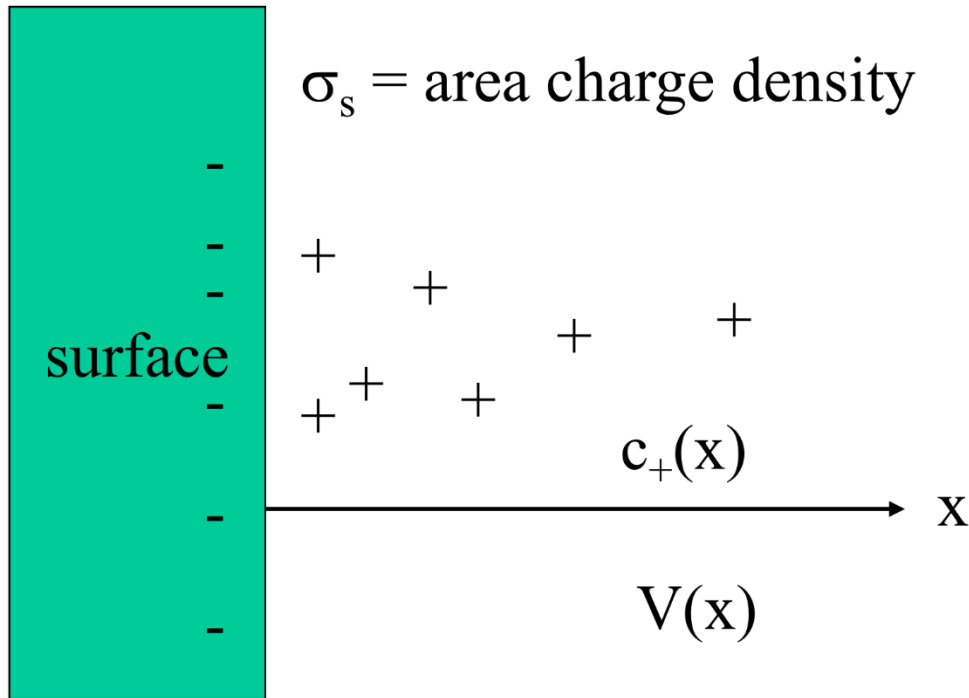
A POTASSIUM CURRENT





- Generation of a membrane voltage does not result in large-scale changes in ion concentration.
- Charge imbalance is localized to thin region near cell membrane

How thick is the charge separation layer?



$$c_+(x) = \frac{1}{2\pi l_B (x + \chi)^2}$$

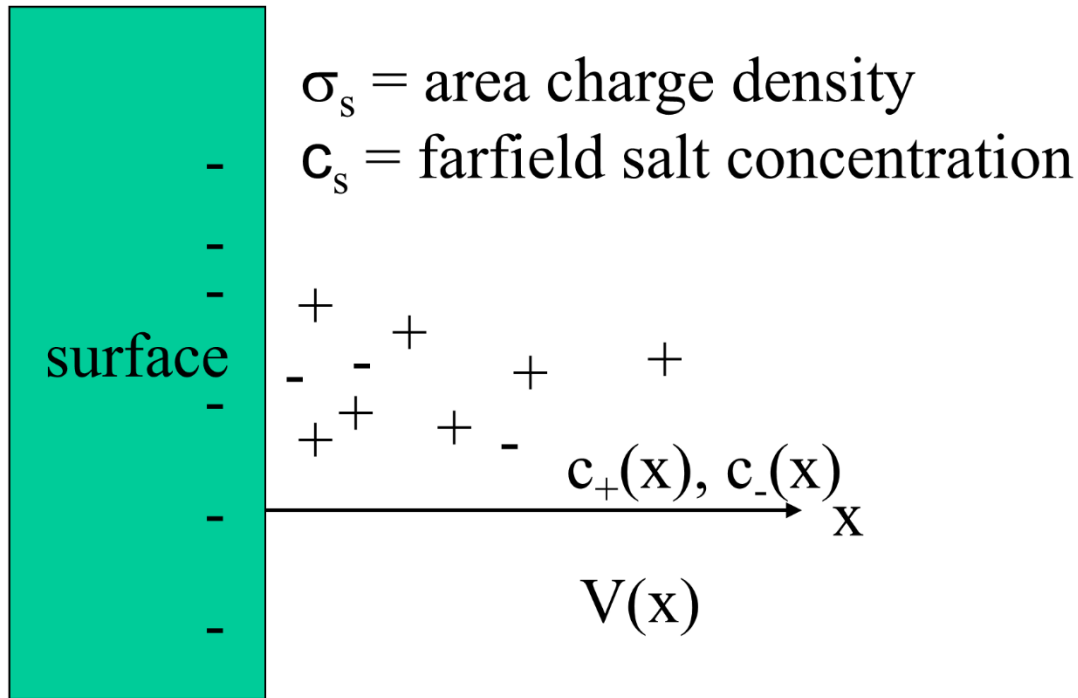
$$\chi = \frac{2\epsilon k_B T}{ne\sigma_s}$$

$$l_B \equiv \frac{q^2}{4\pi\epsilon k_B T}$$

Poisson-Boltzmann equation

- Poisson: charge distribution determines potential
- Boltzmann: potential determines charge distribution
- solve simultaneously

How thick is the charge separation layer?



$$\bar{V} = \bar{V}_0 \exp\left(-\frac{x}{l_D}\right)$$

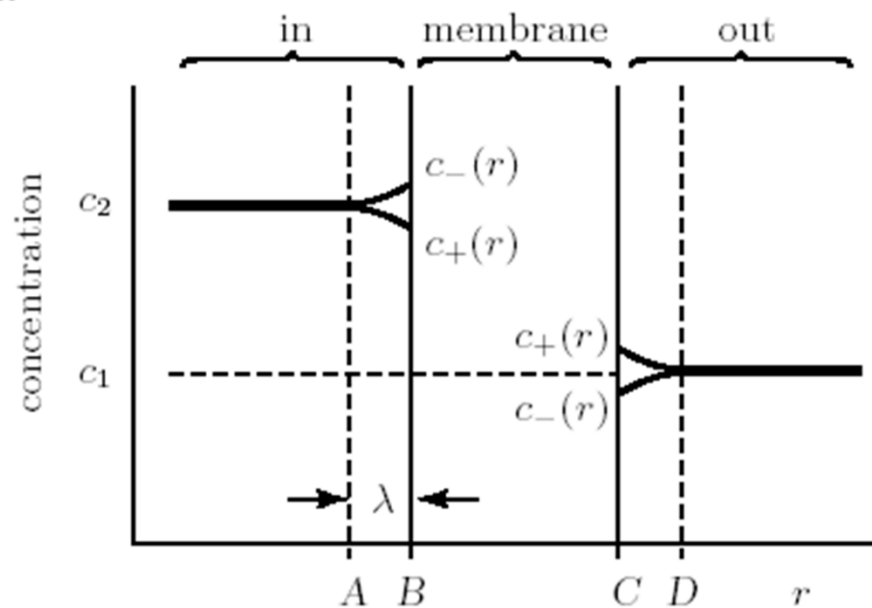
$$l_D = (8\pi l_B c_s)^{-1/2}$$

$$l_B \equiv \frac{q^2}{4\pi\epsilon k_B T}$$

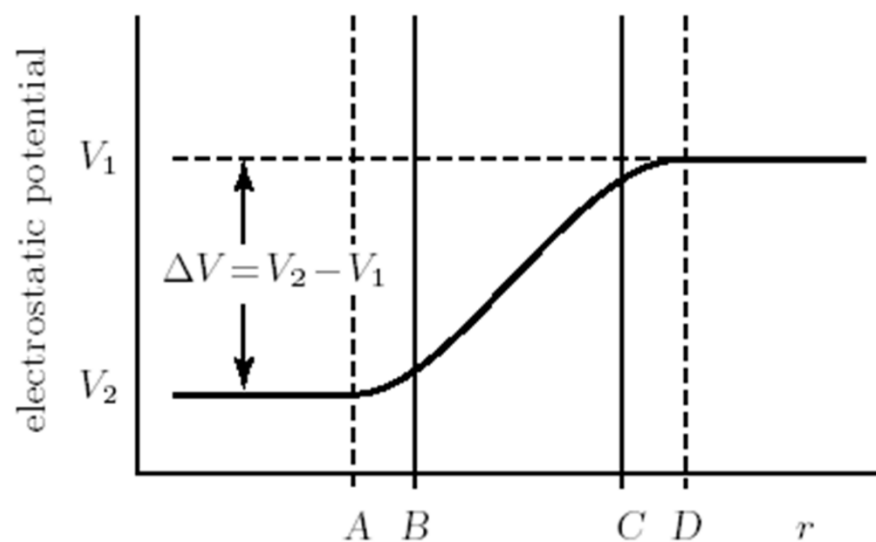
Mix of positive and negative, mobile ions in solution

- \bar{V} : Debye-Huckel form for voltage, for small x .
- Boltzmann: concentration drops exponentially with \bar{V}
- Charged surface is screened over a few l_D
- l_D for 0.2M monovalent salt, body temp, ~ 0.7 nm

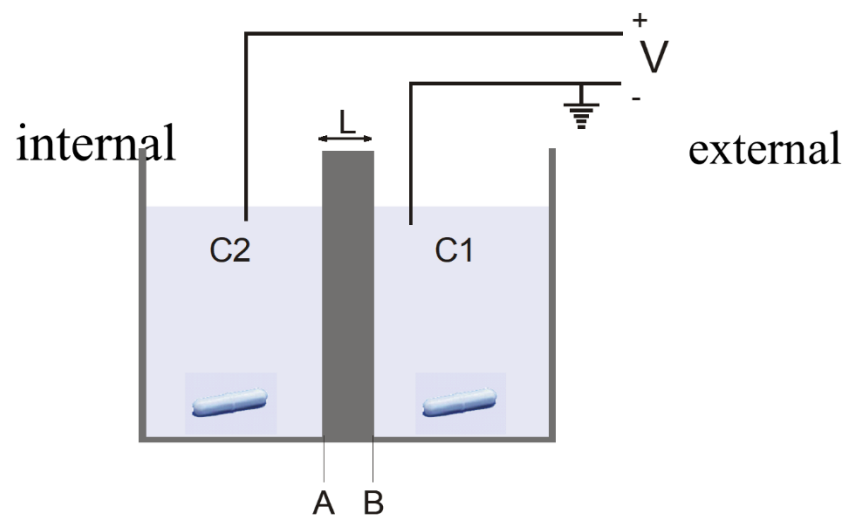
a



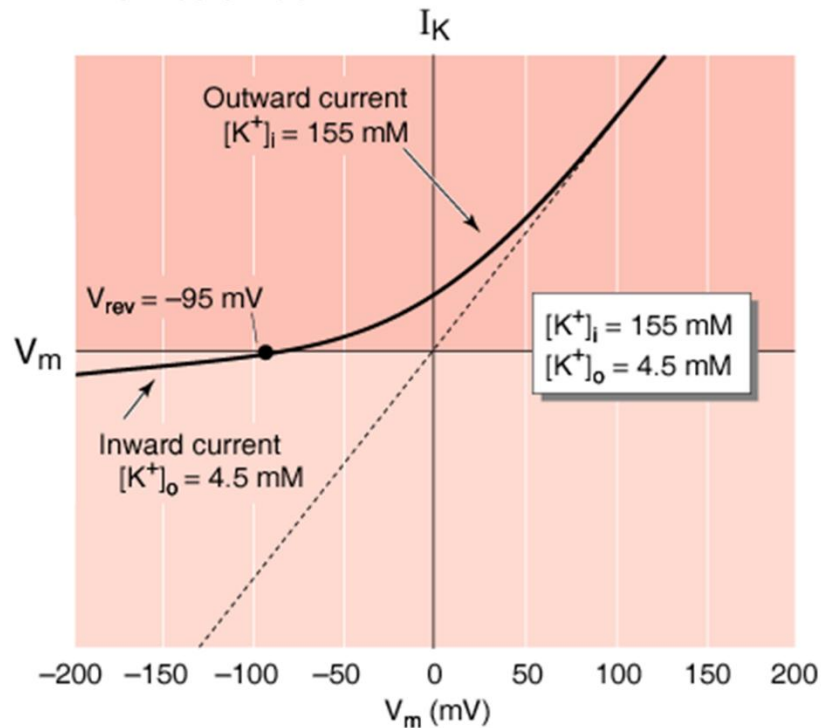
b



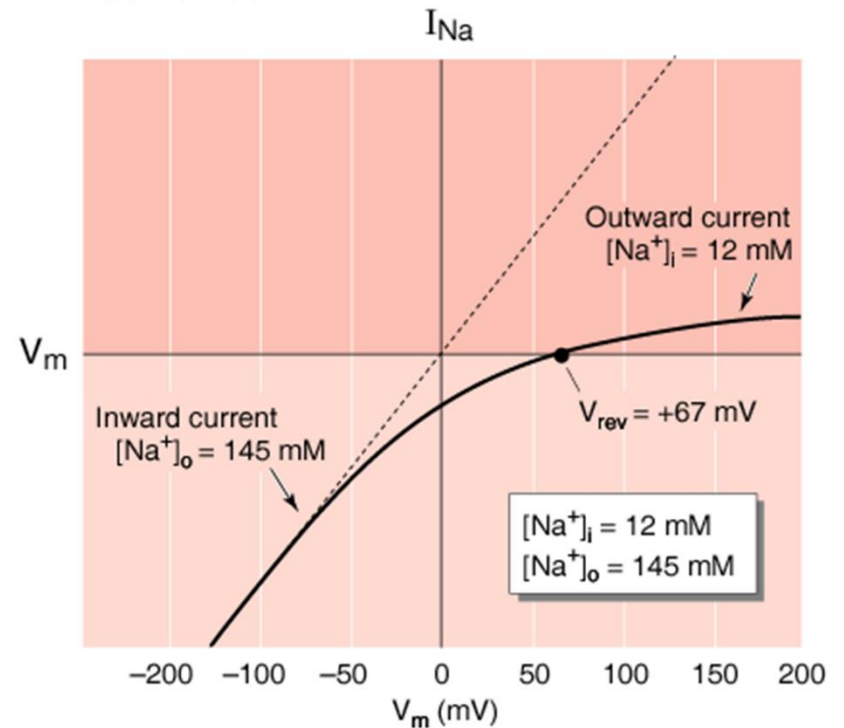
$$V^{Nernst} \equiv -\frac{k_B T}{q} \ln\left(\frac{C_2}{C_1}\right)$$



A POTASSIUM CURRENT



B SODIUM CURRENT



Ion	concentration (mM)		
	interstitial space	cell ("typical")	V^{nernst} (mV) (37°C)
Na^+ ,	145	15	+61
K^+ ,	4.5	120	-85
Cl^- ,	116	20	-47